EE 302

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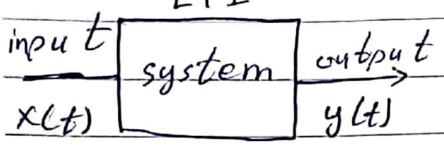
* Graphical Convolution :-



$y(t) = x(t) * h(t)$

$h(t) =$ impulse response

$x(t) * h(t) = h(t) * x(t)$

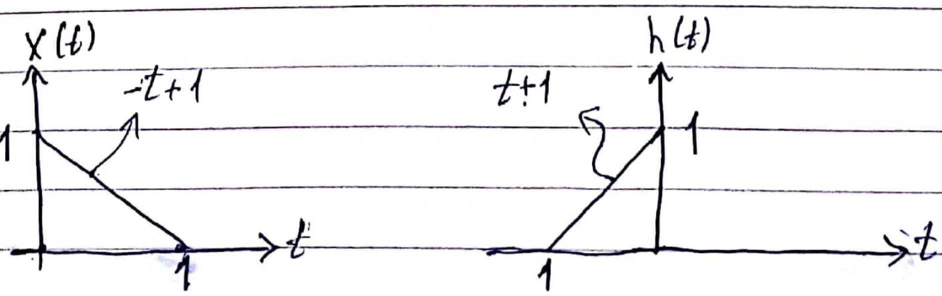
$\boxed{h(t) = y(t)}$ when $\boxed{x(t) = \delta(t)}$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

or

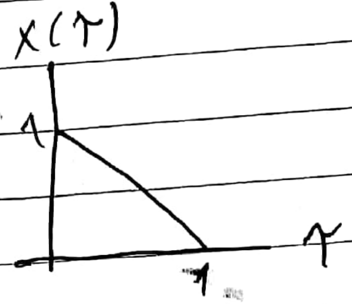
$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Ex for the following signals find the output $y(t)$



$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

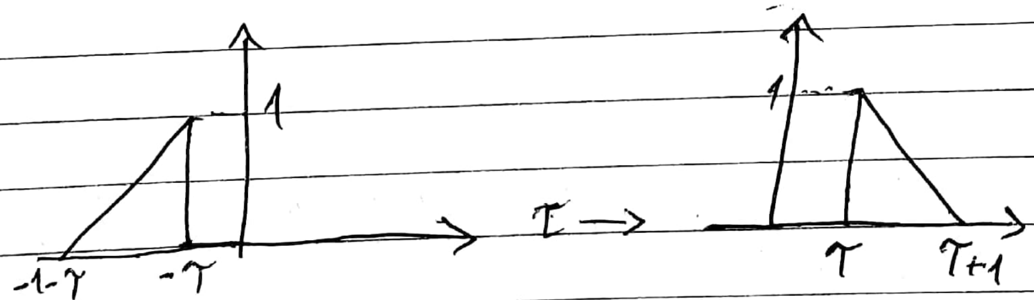


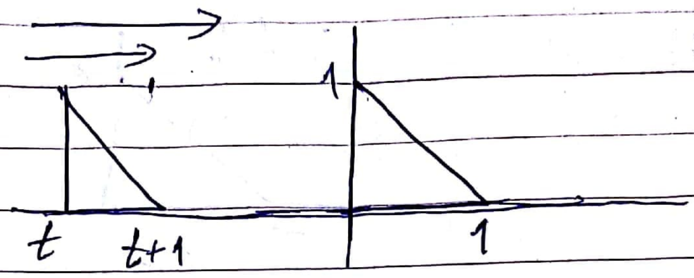
$h(t-\tau) =$

$h(t) \longrightarrow h(\tau) \longrightarrow h(t-\tau)$

$h(-\tau+t)$

$h(t-\tau)$



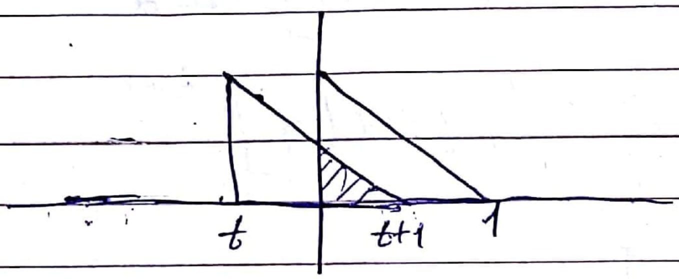


①

$t+1<0 \longrightarrow y(t)=0$

$\boxed{t<-1}$

②



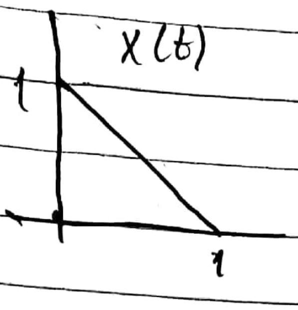
$0<t+1<1$ & $t<0$

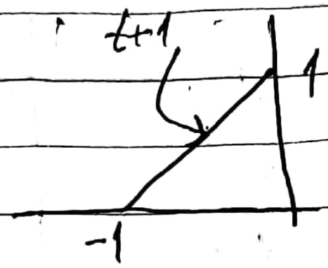
$\boxed{-1<t<0}$

$$y(t)=\int_0^{t+1}[-\tau+1][-\tau+(t+1)]\,d\tau$$

$h(t)=t+1$

$h(t-\tau)=t-\tau+1=-\tau+(t+1)$



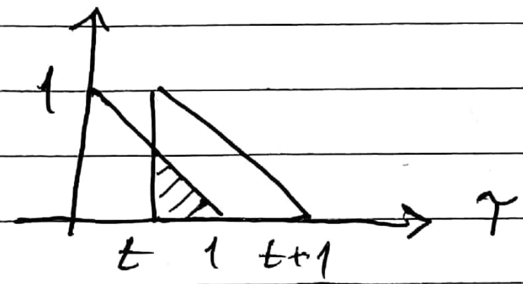


$$h(t-\tau) = t - \tau + 1$$

$$\int_0^{t+1} \left[\tau^2 - (t+1)\,\tau - \tau + (t+1)\right] d\tau$$

$$= \left[\frac{\tau^3}{3} - [t+2]\frac{\tau^2}{2} + [t+1]\,\tau\right]_0^{t+1}$$

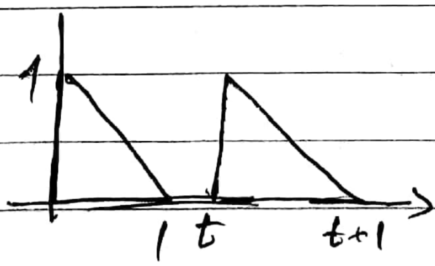
③



$\boxed{0 \leq t \leq 1}$ & $\boxed{t+1 > 1}$

$$y(t) = \left[\frac{\tau^3}{3} - [t+2]\frac{\tau^2}{2} + (t+1)\,\tau\right]_t^1$$

④



$\boxed{t > 1}$

$y(t) = 0$

⇒ Response of the system :

total Response = Zero input response + Zero state response

$$x(t) \rightarrow \overset{\text{LTI}}{\boxed{\text{system}}} \rightarrow y(t)$$

$$y(t) = y_h(t) + y_p(t)$$

$y_h(t) \longrightarrow$ when $x(t) = 0$

$y_p(t) \longrightarrow$ when $x(t) \neq 0$

Ex : find the overall system response for the following system equation : $f(t) = 6t^2$

$y(0) = \frac{25}{18}$ $\quad y'(0) = \frac{2}{3}$

$$y''(t) + 5y'(t) + 6y(t) = f'(t) + f(t)$$

$$\frac{d}{dt} = (\quad)' = D \qquad \frac{d^2}{dt^2} = (\quad)'' = D^2$$

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$D^2 y(t) + 5Dy(t) + 6y(t) = Df(t) + f(t)$

$[D^2 + 5D + 6]\, y(t) = [D+1]\, f(t)$

Zero input response :

$f(t) = 0 \qquad [D^2 + 5D + 6]\, y(t) = 0$

$D^2 + 5D + 6 = 0$

$\lambda^2 + 5\lambda + 6 = 0$

$(\lambda + 3)(\lambda + 2) = 0$

$\lambda_1 = -3 \;, \quad \lambda_2 = -2$

$y_n(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} = C_1 e^{-3t} + C_2 e^{-2t}$

~~Response of the system~~

Zero state response :

for $f(t) = 6t^2$

$y_p(t) = k_0 + k_1 t + k_2 t^2$

$y_p'(t) = k_1 + 2k_2 t$

$y_p''(t) = 2k_2$

$2k_2 + 5[k_1 + 2k_2 t] + 6[k_0 + k_1 t + k_2 t^2] = 12t + 6t^2$

$6k_2 t^2 + [10k_2 + 6k_1]t + [2k_2 + 5k_1 + 6k_0] = 6t^2 + 12t + 0$

$6k_2 = 6 \longrightarrow \boxed{k_2 = 1}$

$10k_2 + 6k_1 = 12$

$10 + 6k_1 = 12$ $\boxed{k_1 = \frac{1}{3}}$

$2k_2 + 5k_1 + 6k_0 = 0$

$2(1) + \frac{5}{3} + 6k_0 = 0$

$6k_0 = -\frac{11}{3} \longrightarrow k_0 = -\frac{11}{18}$

$y_p(t) = -\frac{11}{18} + \frac{1}{3}t + t^2$

$y(t) = y_h(t) + y_p(t)$

$$y(t) = c_1 e^{-3t} + c_2 e^{-2t} - \frac{11}{18} + \frac{1}{3}t + t^2$$

$$y(0)^t = \frac{25}{18} \quad , \quad y'(0)^t = -\frac{2}{3}$$

$$\frac{25}{18} = c_1 + c_2 - \frac{11}{18}$$

$$c_1 + c_2 = \frac{25+11}{18} \qquad c_1 + c_2 = 2$$

$$y'(t) = -3c_1 e^{-3t} - 2c_2 e^{-2t} + \frac{1}{3} + 2t$$

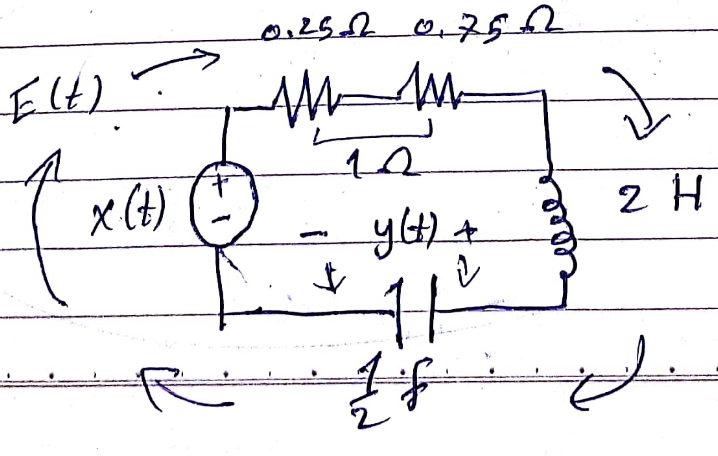
$$-\frac{2}{3} = -3c_1 - 2c_2 + \frac{1}{3} \rightarrow +3c_1 + 2c_2 = \cancel{?}\ +1$$

$$\boxed{c_1 = -3} \qquad \boxed{c_2 = 5}$$

$$y(t) = -3e^{-3t} + 5e^{-2t} - \frac{11}{18} + \frac{1}{3}t + t^2$$

Ex:

find the total response of the following system assuming that $y(0) = 4$ $y'(0) = 5$ and $x(t) = t^2 - 1$



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$$x(t) = V_R + V_L + V_C$$

$$V_R + V_L + V_C = x(t)$$

$$Ri(t) + L\frac{di(t)}{dt} + y(t) = x(t)$$

$$y(t) = \frac{1}{c}\int i(t)\,dt$$

$$\frac{dy(t)}{dt} = \frac{1}{c}\, i(t) \rightarrow \boxed{i(t) = c\,\frac{dy(t)}{dt}}$$

$$Rc\frac{dy(t)}{d(t)} + L\frac{d}{dt}\left[c\,\frac{dy(t)}{dt}\right] + y(t) = x(t)$$

$$\frac{Rc}{Lc}\,\frac{dy(t)}{dt} + \frac{Lc}{Lc}\,\frac{d^2y(t)}{dt^2} + \frac{y}{cL}(t) = \frac{x(t)}{Lc}$$

$$y''(t) + \frac{R}{L}\,y'(t) + \frac{1}{Lc}\,y(t) = \frac{1}{Lc}\,x(t)$$

$$y''(t) + 0.5\,y'(t) + y(t) = x(t)$$

$$D^2y(t) + 0.5\,Dy(t) + y(t) = x(t)$$

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Zero input Respones:

$[D^2+0.5D+1]\,y(t)=0$

$\lambda^2+0.5\lambda+1=0$

$$\lambda = \frac{-0.5 \pm \sqrt{0.5^2-4(1)(1)}}{2(1)}$$

$\lambda_1 = -0.25 + j0.96$

$\lambda_2 = -0.25 - j0.96$

$y_h(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$

$y_h(t) = C_1 e^{(-0.25+j0.96)} + C_2 e^{(-0.25-j0.96)}$

$x(t) = t^2-1$

$y_p(t) = k_0 + k_1 t + k_2 t^2$

$y_p'(t) = k_1 + 2k_2 t$

$y_p''(t) = 2k_2$

$2k_2 + 0.5\,k_1 + k_2 t + k_0 + k_1 t + k_2 t^2 = t^2-1$

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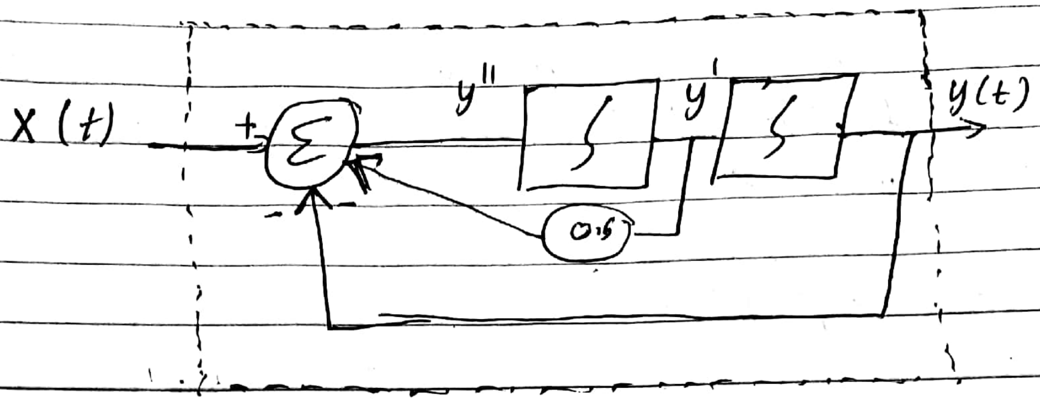
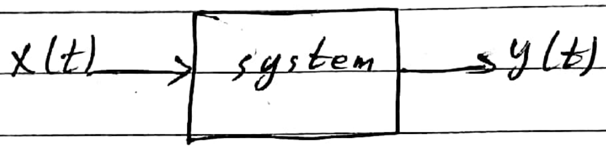
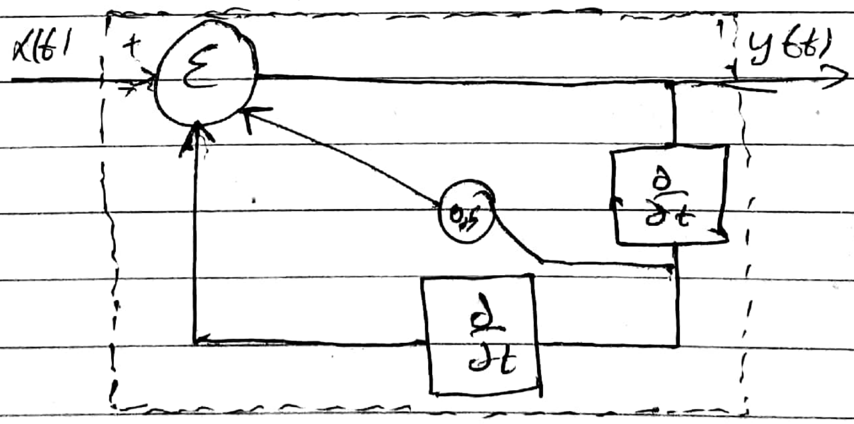
$y''(t) + 0.5\,y'(t) + y(t) = x(t)$

$y''(t) = x(t) - 0.5\,y'(t) - y(t)$





$y(t) = x(t) - y''(t) - 0.5\,y'(t)$



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$3\delta(t) - 5\delta(t-1)$

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* stability :

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

the system is called stable system if the system is BIBO ((Bounded Input Bounded output))

$$y(t) < \infty$$

$$\boxed{Q(D)\, y(t) = p(D)\, x(t)}$$

The values that take $Q(\omega) = 0$ is called "poles"

The values that take $p(\omega) = 0$ is called "Zeros"

$$y(t) = x(t) * h(t)$$

$$Y(S) = X(S)\, H(S)$$

$$H(S) = \frac{Y(S)}{X(S)} \begin{matrix} = 0 \\ = 0 \end{matrix}$$

zeros

poles

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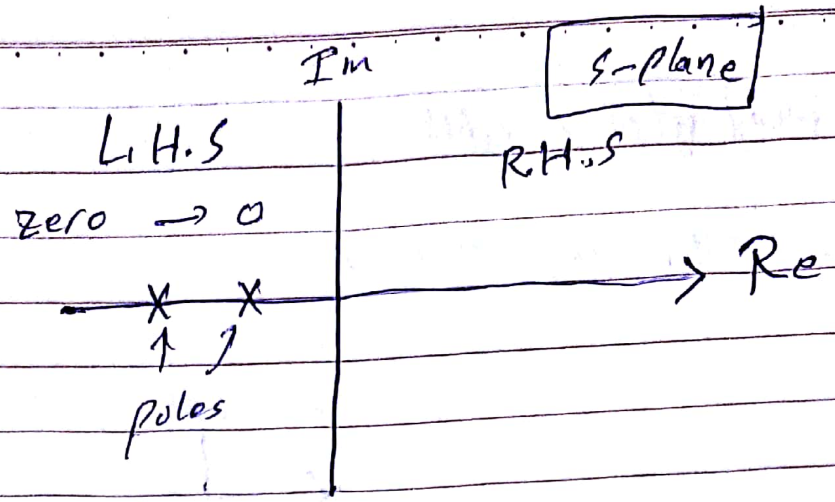

stability . $Q(D) = 0$

يكون النظام مستقر اذا كان:

1] كل الجذور [Poles] موجودة (L.H.S)

$$\boxed{Re\{\lambda\} < 0}$$

2) اذا كان يوجد هناك جذور موجودة على محور المادلات [Im] بشرط انها غير متكررة ولا يوجد جذور في [R.H.S] فان النظام يكون ~~(Unstable)~~
(Critical stable)

3) في حالة وجود ولو جذر واحد في R.H.S فان النظام (Unstable)

4) في حالة تكرار الجذور عند محور المادلات فان النظام (Unstable)

Ex: $y''(t) + 4y'(t) + 40\, y(t) = x(t)$

$Q(D)\, y(t) = p(D)\, x(t)$

$D^2 y(t) + 4D\, y(t) + 40\, y(t) = x(t)$

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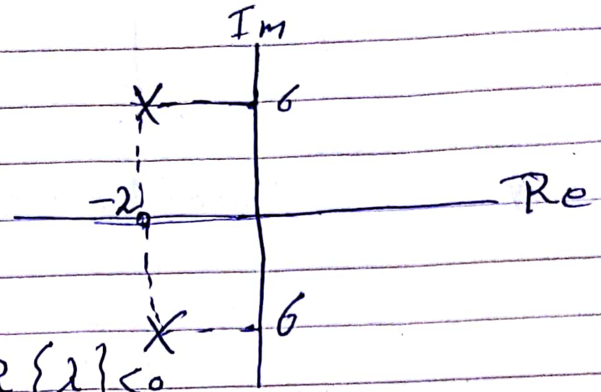
$[D^2 + 4D + 40]\, y(t) = x(t)$

$\therefore Q(D) = D^2 + 4D + 40 \quad , \quad P(D) = 1$

$Q(D) = 0$

$\boxed{\lambda^2 + 4\lambda + 40 = 0}$



$\lambda_1 = -2 + 6i$
$\lambda_2 = -2 - 6i$

$R\{\lambda\} < 0$

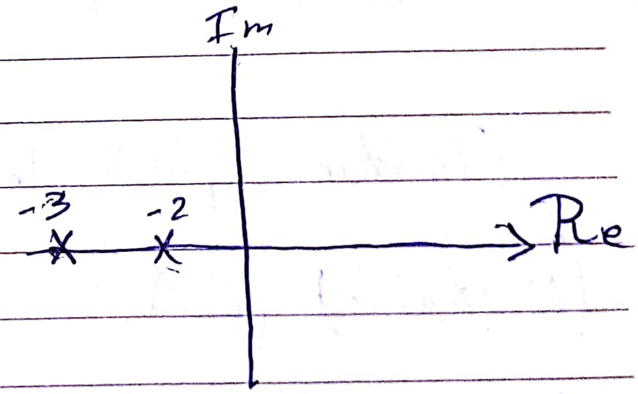
system is stable

2] $(D^2 + 5D + 6)\, y(t) = (D+1)\, f(t)$

$D^2 + 5D + 6 = 0$

$\lambda^2 + 5\lambda + 6 = 0$

$\lambda = -2 \qquad \lambda = -3$



stable

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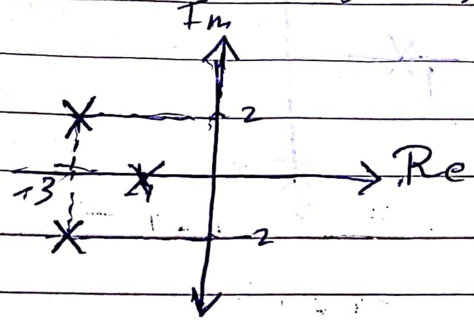
~~2] $(D^2+5D+6)\,y(t) = (D+1)\,f(t)$~~

3] $\underbrace{(D+1)(D^2+4D+8)}_{Q(D)}\,y(t) = (D-3)\,x(t)$

$(\lambda+1)=0 \qquad \lambda_1=-1$

$\lambda^2+4\lambda+8=0 \qquad \lambda_2=-2+j2$

$\lambda_3=-2-j2$



stable

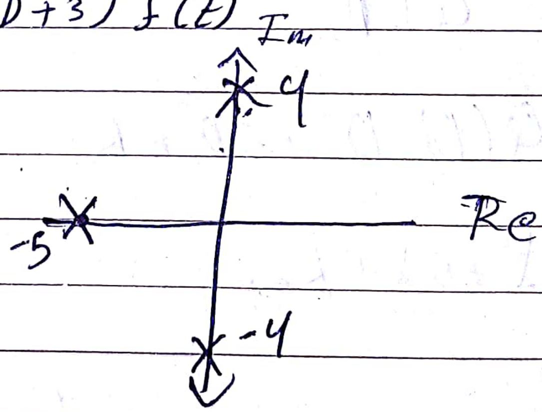
4] $(D-2)(D+3)(D^2+4D+8)\,y(t) = (D^2+1)\,x(t)$

$\lambda=2 \qquad$ Unstable

5] $\underbrace{(D+5)(D^2+16)}_{Q(D)}\,y(t) = (D^2+2D+3)\,f(t)$

$\lambda+5=0 \qquad \lambda_1=-5$

$\lambda^2+16=0 \qquad \lambda_2=\pm 4i$



Critical stable

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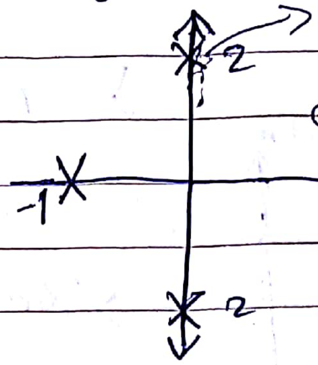
6] $\underbrace{[D+5][D^2+4]^2}_{Q(D)} y(t) = f(t)$

$\lambda + 1 = 0 \longrightarrow \lambda = -1$ $\qquad [D^2+4]^2 = 0$

$[\lambda^2+4][\lambda^2+4] = 0$

$\lambda_1 = \pm 2i \qquad \lambda_2 = \pm 2i$



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Unstable

7] find the range of k so that the system is stable

$\underbrace{[D^2+5D+k]}_{Q[D]} y(t) = D x(t)$

$Q(D) = D^2 + 5D + k$

$\lambda^2 + 5\lambda + k = 0$

$$\lambda_{1,2} = \frac{-5 \pm \sqrt{25-4k}}{2}$$

for stable system

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$$Re\left[\frac{-5 \pm \sqrt{25-4k}}{2}\right] < 0$$

$$-5 \pm \sqrt{25-4k} < 0$$

$$\pm \sqrt{25-4k} < 5$$

$$+ \sqrt{25-4k} < 5$$

$$-\sqrt{25-4k} < 5 \qquad \sqrt{25-4k} < 5$$

$$25-4k > 0 \longrightarrow 25 > 4k$$

$$k < \frac{25}{4} \qquad k > 0$$

$$0 < k < \frac{25}{4}$$

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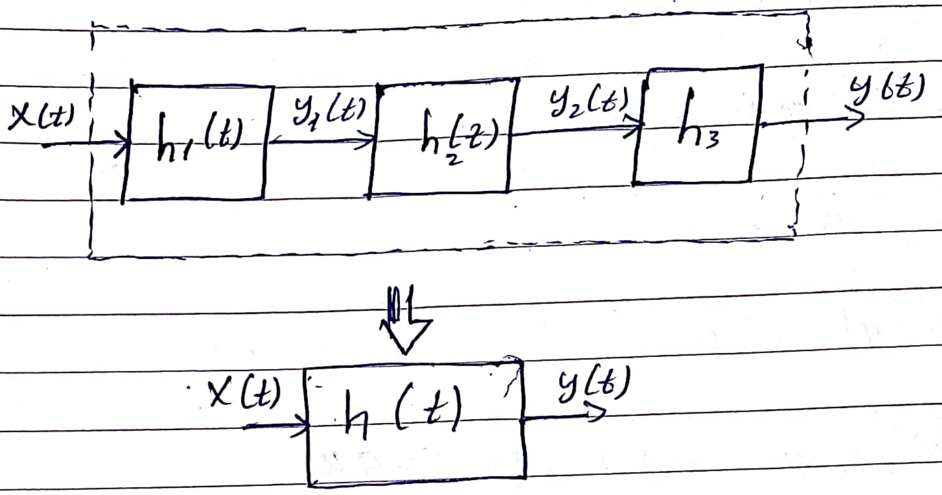
overall Impulse response of the system :-

1] for casecode system (series)



$$y_1(t) = \underbrace{x(t) * h(t)}$$

$$y_2(t) = y_1(t) * h_2(t) = x(t) * \underbrace{h_1(t) * h_2(t)}$$

$$y(t) = y_2(t) * h_3(t)$$

$$y(t) = x(t) * \overbrace{h_1(t) * h_2(t) * h_2(t)}^{h(t)}$$

2] for Casecoded system (parallel)

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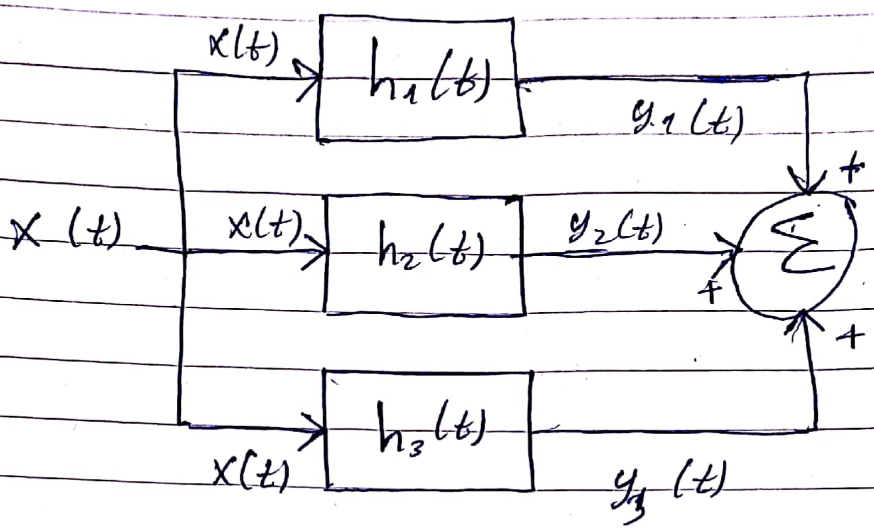


$y_1(t) = x(t) * h_1(t)$

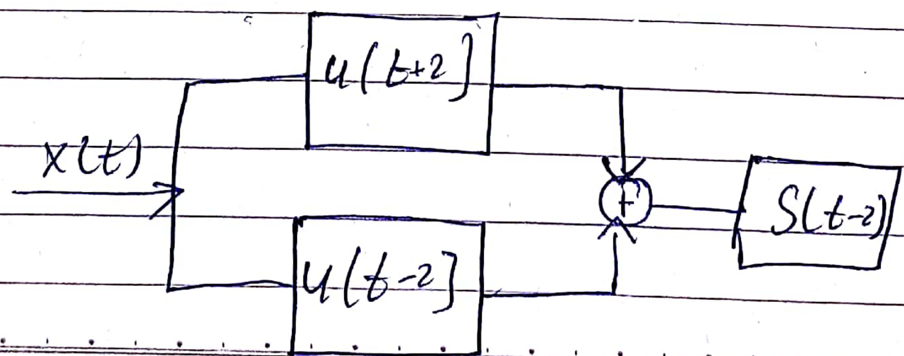
$y_2(t) = x(t) * h_2(t)$

$y_3(t) = x(t) * h_3(t)$

$y(t) = y_1(t) + y_2(t) + y_3(t)$

$y(t) = x(t) * \left[h_1(t) + h_2(t) + h_3(t)\right]$

Ex: find and skeatch the over all Impulse response of the system:-



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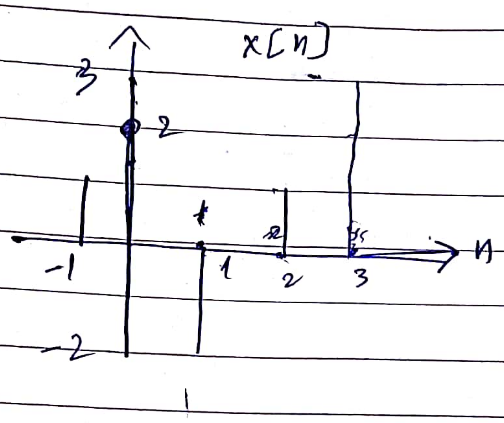
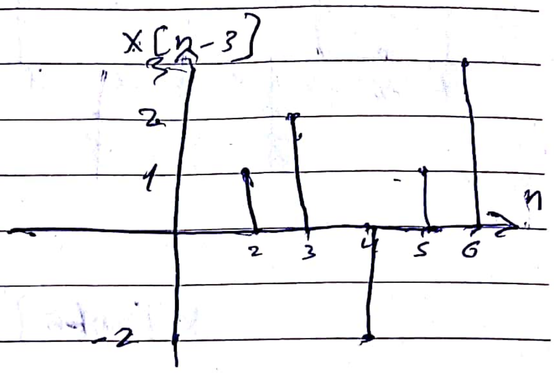
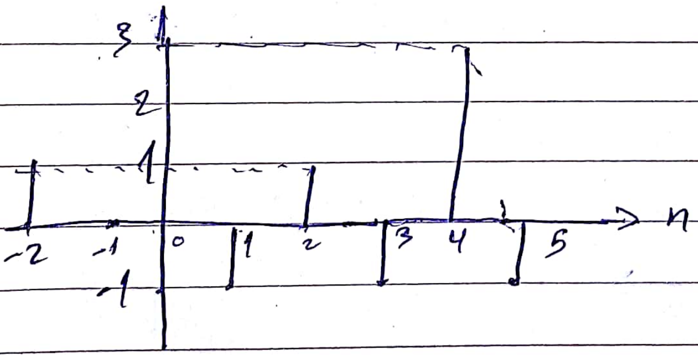
Ex : Given $x[n]$ as shown below, find $x[n-3]$, $x[n+1]$, $x[n-5]$





Ex: Given $x[n]$ as shown below, find

$3x[-3n+5]$



$x[n] \xrightarrow{n \to n+5} x[n+5]$

$x[-3n+5] \xleftarrow{n \to -n} x[3n+5]$

$\xrightarrow{\times 3} 3x[-3n+5]$

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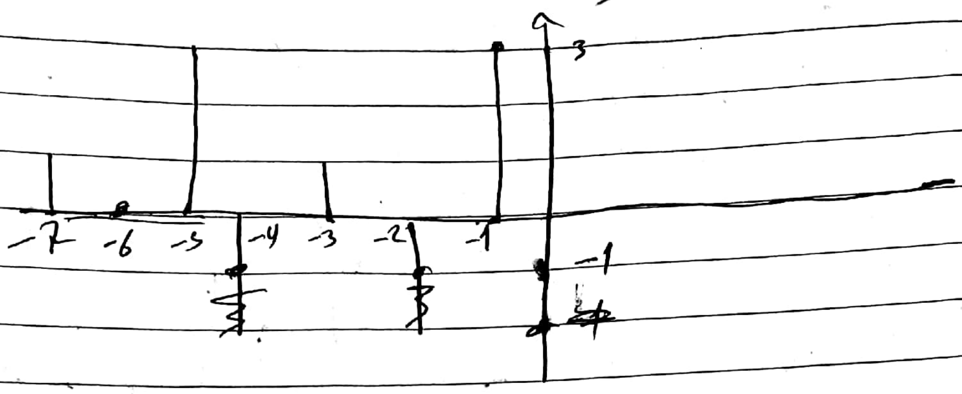
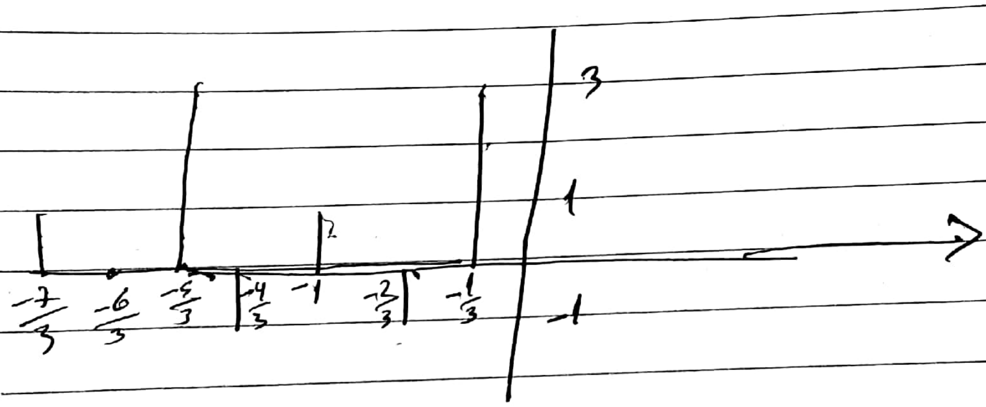
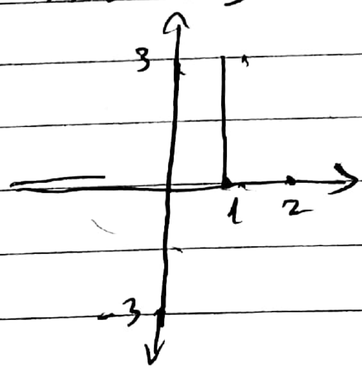
$x[n+5]$



$x[3n+5]$



$3x[3n+5]$



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Descrete time systems:- τ

in continuous time system

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

in descrete time system:-

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

⇨ classification of descrete time system:

1) linear or nonlinear

2) causal or Non causal

3) Memory less or Memory

4) dynemic or non dynamic

5) stable or Non stable

6) Time varient or time inverent

Ex: $y[n] = 5x[n-2]+7$

$x_1[n] \longrightarrow y_1[n] = 5x_1[n-2]+7$

$x_2(n] \longrightarrow y_2(n] = 5x_2[n-2]+7$

$x_1(n]+x_2[n] \longrightarrow$

$y_2[n] = 5[x_1[n-2]+x_2(x-2]]+7$

$y_3(n] \Rightarrow y_1(n)+y_2(n)$

Non linear system

$n=0$ $y(0] = 5x[-2]+7$

Time output $\geq$ Time input causal system

Time output $\neq$ Time input $\boxed{\text{Memory system}}$

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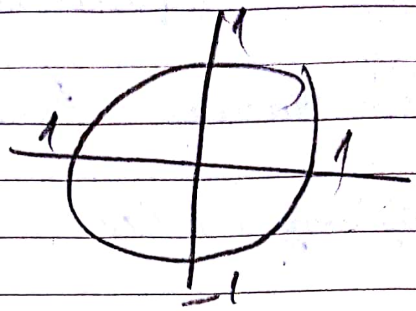
$|\delta| \leq 1$

$x[n] = \beta < \infty$



$y[n] = 5\beta + 2 < \infty$

stable system [BIBO]

$y[n-k] = 5x[n-k-2] + 7$

$x[n-k] \rightarrow \hat{y}[n] = 5x[n-k-2] + 7$

Time Inverent system

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Defference equation :

in continuous time signal [Deffirentiel equation]

$y'' + 5y' + 6y = x(t)$

$\underbrace{[D^2 + 5D + 6]}_{Q(D)} y(t) = x(t)$

$Q(D)\, y(t) = P(D)\, x(t)$

in descrete - time signal. Defference eqaation

$Q[E]\, y[n] = P[E]\, x[n]$

in Continuon time systems

$y_h = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \ldots$

$= C_1 [e^{\lambda_1}]^t + C_2 [e^{\lambda_2}]^t + \ldots$

in descrete time system :

$y_h(n) = C_1 \gamma^n + C_2 \gamma_2^n + C_3 \gamma_3^n + \ldots$

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Ex: solve the following difference equations:

1) $\boxed{y[n+1] - 6y[n] = 0}$ $\qquad y[0] = 3$

$Ey(n) - 6y[n] = 0$ $\qquad y[n+3]$

$[E-6]\, y[n] = 0$ $\qquad E^3 y[n]$

$\gamma - 6 = 0 \longrightarrow \gamma = 6$

$y[n] = c\gamma^n$

$y[n] = c(6)^n$

$3 = c[6]^0$

2] $y(n-3] - y[n-4) = 0$

$y(N+1) - y(N) = 0$

$Ey[N] - y[N] = 0$

$[E-1]\, y(N) = 0$

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$E-1=0$ $\qquad$ $\gamma-1=0 \longrightarrow \gamma=1$

$y[N]=c(1)^N$

$N=n-4$ $\qquad$ $n=N+4$

$y[n-4]=c(1)^{n-4}$

$1+N=n-4+1$

$N+1=n-3$

$y[5-4]=c(1)^{5-4}$

$y(1)=c(1)=10$

$c=10$

3] $y[n+2]-3y[n+1]+2y[n]=x[n]$

$x[n]=n$

$y[n+2]-3y[\cancel{\ }n+1]+2y[n]=0$

$E^2 y[n]-3Ey[n]+2y[n]=0$

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$[E^2 - 3E + 2]\, y(n) = 0$

$\gamma^2 - 3\gamma + 2 = 0$

$(\gamma - 2)(\gamma - 1) = 0$

$\gamma_1 = 2 \quad \gamma_2 = 1$

$y[n] = c_1 \gamma_1^n + c_2 \gamma_2^n$

$k_1 n + k_2 + k_{2n} = n$

$k_1 + k_2 = 1$

$y_h[n] = c_1 (2)^n + c_2 (1)^n$

$y_p(n) = k_0 + k_1 n$

$y_p[n+2] = k_0 + k_1[n+2]$

$y_p(n+1) \; k_0 + k_1[n+1]$

$k_0 + k_1[n+2] - 3[k_0 + k_1(n+1)] + 2[k_0 + k_1 n] = n$

$k_0 + k_1 n + 2k_1 - 3k_0 - 3k_1 n - 3k_1 + 2k_0 + \cancel{2k_1} + 2k_1 n = n$

$k_0 + 2k_1 - 3k_0 - 3k_1 + 2k_0 = 0$

$-k_1 = 0 \qquad k_1 = 0$

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$k_1 - 3k_1 + 2k_1 = 1$

Particular solution:-

$x[n]$	form of
γ^n	$\beta\gamma^n$
e^{kt}	βe^{kt}
n^k	$\beta_0 + \beta_1 n^1 + \beta_2 n^2 + \beta_3 n^3 + \ldots\ldots \beta_k$
t^K	$\beta_0 + \beta_1 t + \beta_2 t^2 + \ldots$
$\cos(kn)$	$\beta_1 \cos(kn) + \beta_2 \sin(kn)$
$\cos(kt)$	$\beta_1 \cos(kt) + \beta_2 \sin(kt)$

4] $y[n+2] - y[n+1] - 6y[n] = 36n$

1] Zero input response:-

$y[n+2] - y[n+1] - 6y[n] = 0$

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$E^2 y[n] - Ey[n] - 6y[n] = 0$

$[E^2 - E - 6] y[n] = 0$

$(\gamma - 3)(\gamma + 2) = 0$

$\gamma_1 = 3 \qquad \gamma_2 = -2$

$y_h(n) = c_1 \gamma_1^n + c_2 \gamma_2^n$

$y_n(n) = c_1 3^n + c_2 (-2)^n$

$y_p[n] = \beta_0 + \beta_1 n$

$E y_p[n] = \beta_1$

$E^2 y_p[n] = 0$

$0 - \beta_1 - 6[\beta_0 + \beta_1 n] = 36n$

$-\beta_1 - 6\beta_0 - 6\beta_1 n = 36n$

$6\beta_1 = 36 \qquad \beta_1 = -6$

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$$E^2 y[n] - E y[n] - 6 y[n] = 0$$

$$[E^2 - E - 6]\, y[n] = 0$$

$$(\gamma - 3)(\gamma + 2) = 0$$

$$\gamma_1 = 3 \qquad \gamma_2 = -2$$

$$y_h[n] = c_1 \gamma_1^n + c_2 \gamma_2^n$$

$$y_h[n] = c_1 3^n + c_2 [-2]^n$$

$$y_p[n] = \beta_0 + \beta_1 n$$

$$E y_p[n] = \beta_1$$

$$E^2 y_p[n] = 0$$

$$0 - \beta_1 - 6[\beta_0 + \beta_1 n] = 36n$$

$$-\beta_1 - 6\beta_0 - 6\beta_1 n = 36n$$

$$6\beta_1 = 36 \qquad \beta_1 = -6$$

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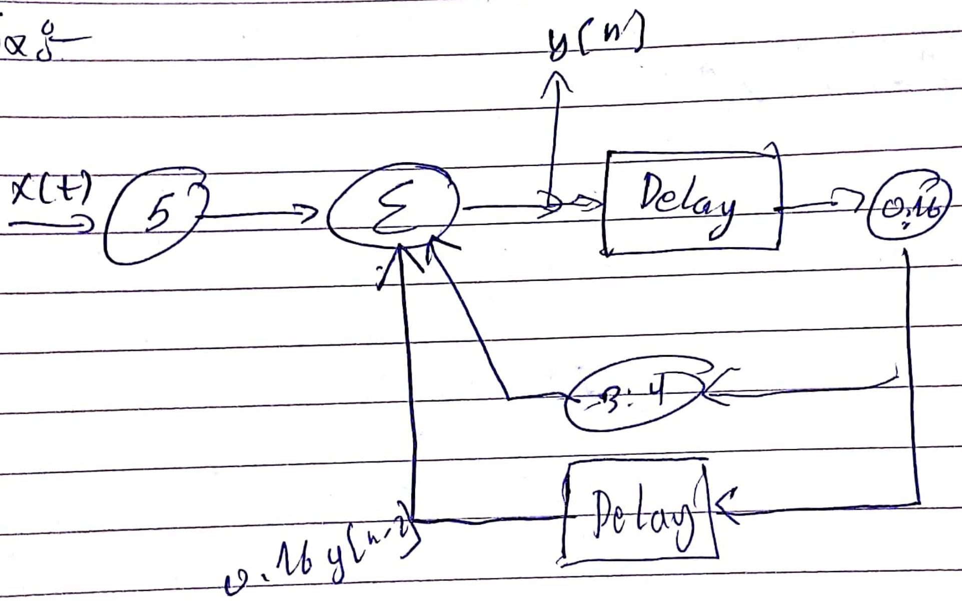
$-\beta_1 - 6\beta_0 = 0$

$6 - 6\beta_0 \to \qquad \beta_0 = 1$

$y_p[n] = 1 - 6n$

$y[n] = y_h[n] + y_p[n] = C_1 3^n + C_2 [-2]^n - 1 - 6n$

Ex:



$y[n] = 5x(n) + 0.16y[n-2] + 0.6y[n-1]$

$y[n+2] - 0.6y(n+1) - 0.16y[n] = 5x[n+2]$

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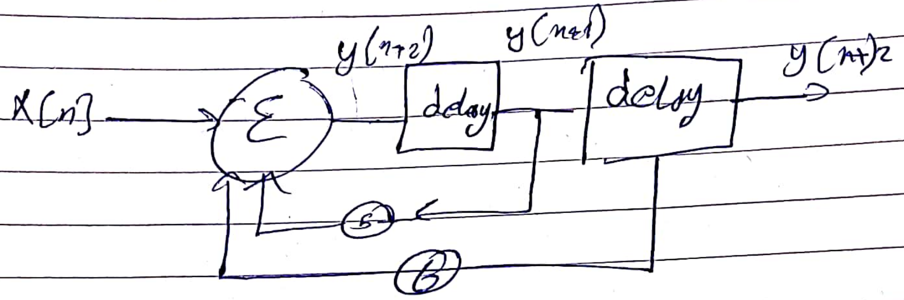
Ex : $y[n+2] - 5y[n+1] + 6y[n] = x[n]$

$x[n] = 3\sin(4/3\, n)$

$y[n+2] = 5y[n+1] - 6y[n] + x[n]$



Zero Input Response :

$y[n+2] - 5y[n+1] + 6y[n] = 0$

$E^2 y[n] - 5Ey[n] + 6y[n] = 0$

$[E^2 - 5E + 6]\, y[n] = 0$

$\gamma^2 - 5\gamma + 6 = 0$

$(\gamma - 2)(\gamma - 3) = 0$

$\gamma = 2 \qquad \gamma = 3$

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$y[n] = c_1 \gamma_1^n + c_2 \gamma_2^n$

$y_h[n] = c_1 2^n + c_2 3^n$

$y_p[n] = \beta_1 \sin[4n] + \beta_2 \cos[4n]$

$y_p[n+1] = 4\beta_1 \cos[4n] - 4\beta_2 \sin[4n]$

$-16\beta_1 \sin[4n] - 16\beta_2 \cos(4n) - 20\beta_1(4n) + 20$

$\beta_2 \sin[4n] + 6\beta_1 \sin(4n) + 6\beta_2 \cos(4n) = 3\sin[4n]$

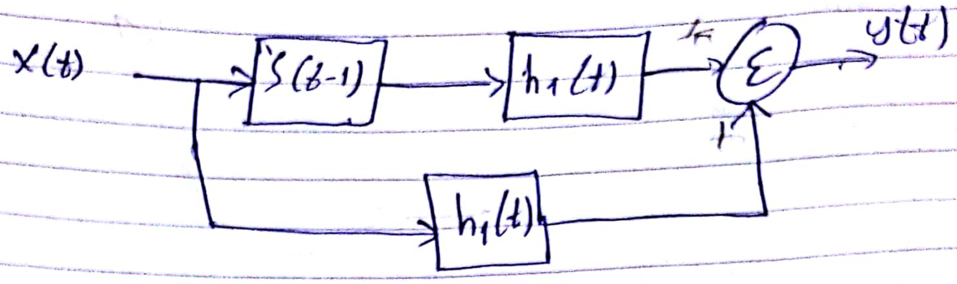
$-16\beta_1 + 20\beta_2 + 6\beta_1 = 3$

$\boxed{-10\beta_1 + 20\beta_2 = 3}$

$-16\beta_2 - 20\beta_1 + 6\beta_2 = 0$

$\boxed{-20\beta_1 - 10\beta_2 = 0}$

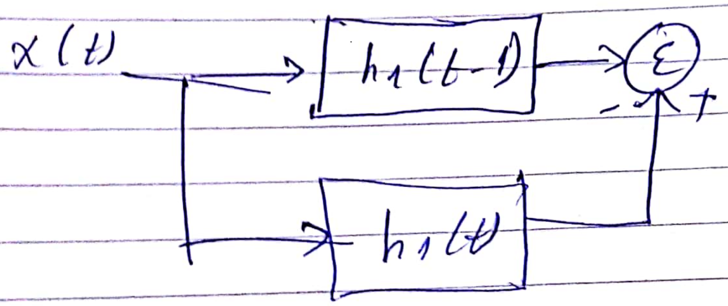
$y[n] = c_1 2^n + c_2 3^n + \beta_1 \sin[4n] + \beta_2 \cos(4n)$

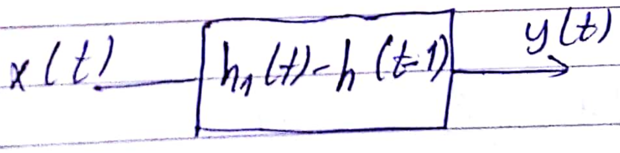


$h_1(t) = 5u(t) - 5u(t-1)$

$x(t) = 3\delta(t) + \delta(t-1)$

find $h(t)$, $y(t)$





$h_1(t-1) = 5u(t-1) - 5u(t-2)$

$h(t) = h_1(t) - h_1(t-1)$

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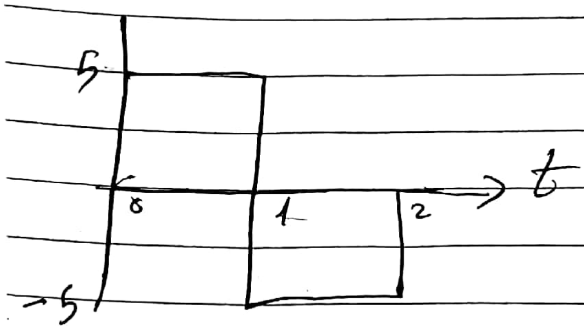
$[5u(t) - 5u(t-1)] - [5u(t-1) - 5u(t-2)]$

$h(t) = 5u(t) - 10u(t-1) + 5u(t-2)$



$y(t) = x(t) * h_1(t)$

$= [3\delta(t) + \delta(t-1)] * h(t)$

$= 3h(t) + h(t-1)$

$= 3[5u(t) - 10u(t-1) + 5u(t-2)]$

$+ \cancel{5u(t} + 5u(t-1) - 10u(t-2) + 5u(t-3)$

$y(t) = 15u(t) - 25u(t-1) + 5u(t-2) + 5u(t-3)$

2] $h_k = 4\left[\frac{1}{5}\right]^k + \left[\frac{1}{2}\right]^k \quad k \geq 0$

$h = y$ when $x = \delta$

$h[n] = y[n]$ when $x[n] = \delta[n]$

$h[k+2] + \beta_1 h[k+1] + \beta_2 h[k] = 0$

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$\delta^2 h[k] + \beta_1 \delta \cdot h[k] + \beta_2 h[5] = 0$

$[\delta^2 + \beta_1 \delta + \beta_2] h(k) = 0$

$\gamma^2 + \beta_1 \gamma + \beta_2 = 0$

$\gamma_1 = \frac{1}{5} \qquad \gamma_2 = \frac{1}{2}$

$[\frac{1}{5}]^2 + \beta_1 [\frac{1}{5}] + \beta_2 = 0$

$\frac{1}{25} + \frac{1}{2}\beta_1 + \beta_2 = 0$

$1 + 5\beta_1 + 25\beta_2 = 0$

$5\beta_1 + 25\beta_2 = -1$

$[\frac{1}{2}]^2 + \beta_1 [\frac{1}{2}] + \beta_2 = 0$

$\frac{1}{4} + \frac{\beta_1}{2} + \beta_2 = 0$

$1 + 2\beta_1 + 4\beta_1 = 0$

$2\beta_1 + 4\beta_2 = -1$

$\beta_1 = -0.7 \quad , \quad \beta_2 = 0.1$

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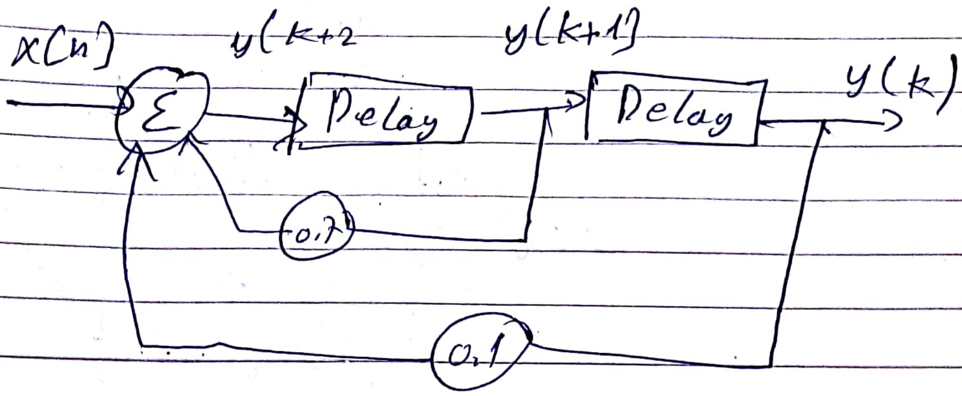
$h[k+2] - 0.7h[k+1] + 0.1h[k] = 0$

$h[k+2] = 0.7\,h[k+1] - 0.1h[k]$



if - $X_k = 2\delta(k-4)$ find $y(k)$

$y\,k = 8\left[\frac{1}{5}\right]^{k-4} + 2\left[\frac{1}{2}\right]^{k-4}$

Ex: Draw the block Diagram & find the total response if

$x(n) = 3^n + 5n^2$

$y[n+2] - 16\,y[n] = x(n)$

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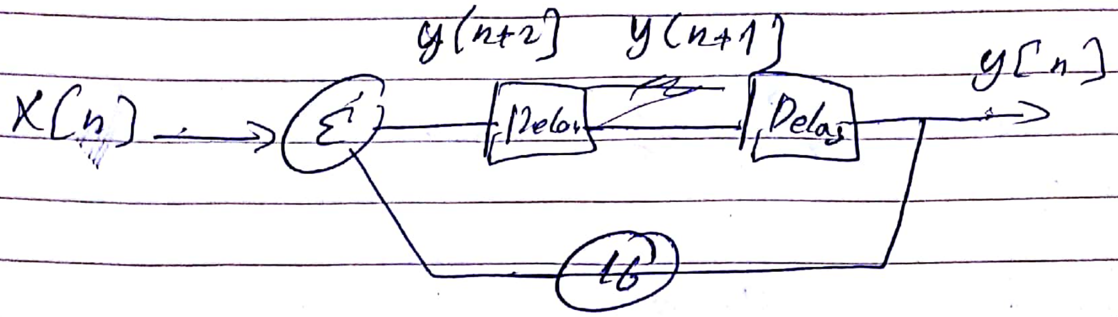
$y[n+2] = 16\, y[n] + x[n]$



$y[n+2] - 16\, y(n) = 0$

$E^2 y(n) - 16\, y(n) = 0$

$[E^2 - 16]\, y(n) = 0$

$\gamma^2 - 16 = 0 \qquad \gamma = 4 \qquad \gamma_2 = -4$

$$\boxed{y_h[n] = c_1 4^n + c_2 (-4)^n}$$

$y_p(n) = \beta_0 3^n + \beta_1 + \beta_2 n + \beta_3 n^2$

$y_p(n+1) = \beta_0 3^n \ln[3] + \beta_2 + 2\beta_3 n$

$y_p(n+2) = \beta_0 3^n [\ln(3)]^2 + 2\beta_3$

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$$\beta_0 3^n [\ln(3)]^2 + 2\beta_3 - 16\beta_0 3^n - 16\beta_1 - 16\beta_2 n - 16\beta_3 n^2$$

$$4\, 3^n + 5n^2$$

$$-16\beta_3 = 5 \qquad \beta_3 = -\frac{5}{16}$$

$$-16\beta_2 = 0 \qquad \beta_2 = 0$$

$$2\beta_3 - 16\beta_1 = 0$$

$$2\left(-\frac{5}{16}\right) - 16\beta_1 = 0$$

$$16\beta_1 = -\frac{10}{16} \qquad \beta_1 = -\frac{10}{16^2}$$

$$\beta_0\{\ln[3]\}^2 - 16\beta_0 = 1$$

$$\beta_0 = \frac{1}{[\ln(3)]^2 - 16}$$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y[n] = x[n] * h[n]$$

$$= \sum_{n=-\infty}^{\infty} x[k]\, h[n-k]$$

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Ex: find y(n) if

x[n] = {0, 1, -1, 5}

h[n] = {-3, 1, 4, -6}

	0	1	-1	5
-3	0	-3	3	-15
1	0	1	-1	5
4	0	4	-4	20
-6	0	-6	6	-30

y[n] = {0, -3, 4, -12, -5, 26, -30}

Ex: x(n) = [1, 2, 3, 1]

h(n) = {0, 3, 4, -1, -2}

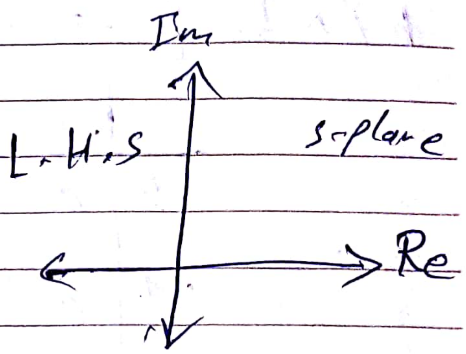
	0	0	1	2	3	1
0	0	0	0	0	0	0
3	0	0	3	6	9	3
4	0	0	4	8	12	4
-1	0	0	-1	-2	-3	-1
-2	0	0	-2	-4	-6	-2

do

$y[n] = \{0, 0, 0, 3, 10, 16, 11, -3, -7, -2\}$

stability :

in continuous times



$Q(D)\, y(t) = P(D)\, x(t)$

$Q(D) = 0$

in descrete time

$Q[E]\, y[n] = P[E]\, x[n]$

$Q(E) = 0$

∴ solution is always γ^n

$0 < \gamma < 1 \xrightarrow{Ex} [0.5]^n$ stable $\quad n = \infty \quad [0.5]^{\infty} = 0$

$\gamma = 1$, stable / $\gamma > 1 \to Ex [2]^n$ unstable